

Exercise 35

Find y'' by implicit differentiation.

$$x^2 + 4y^2 = 4$$

Solution

Differentiate both sides with respect to x .

$$\frac{d}{dx}(x^2 + 4y^2) = \frac{d}{dx}(4)$$

$$\frac{d}{dx}(x^2) + 4\frac{d}{dx}(y^2) = 0$$

$$(2x) + 4\left[(2y) \cdot \frac{d}{dx}(y)\right] = 0$$

$$2x + 8yy' = 0$$

$$x + 4yy' = 0$$

Solve for y' .

$$y' = -\frac{x}{4y}$$

Differentiate both sides with respect to x again to get y'' .

$$\frac{d}{dx}(y') = \frac{d}{dx}\left(-\frac{x}{4y}\right)$$

$$y'' = -\frac{d}{dx}\left(\frac{x}{4y}\right)$$

$$= -\frac{\left[\frac{d}{dx}(x)\right](4y) - \left[\frac{d}{dx}(4y)\right](x)}{(4y)^2}$$

$$= -\frac{(1)(4y) - (4y')(x)}{16y^2}$$

$$= -\frac{y - xy'}{4y^2}$$

$$= -\frac{y - x\left(-\frac{x}{4y}\right)}{4y^2}$$

$$= -\frac{y + \frac{x^2}{4y}}{4y^2}$$

$$= -\frac{\frac{4y^2 + x^2}{4y}}{4y^2} = -\frac{(4 - x^2) + x^2}{4y^2} = -\frac{4}{4y^2} = -\frac{1}{y} = -\frac{1}{4y^3}$$